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COMPLEMENTARY TOTALLY ANTIMAGIC TOTAL GRAPHS

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ABSTRACT

For a graph G, with the vertex set V(G) and the edge set E(G), a total labeling is a bijection f from V (G) U E(G) to the set of integers {1, 2, …, |V (G) |+| E(G) |}. The edge weight sum is f(u)+f(uv)+f(v) = k for every edge uv ϵ E(G) and the vertex weight sum is $f(u)+\sum_{v}$ЄV (G) f(uv) = k1 for vertex vϵV(G) where k and k1 are constants called valences. If k's(k1's) are different, a total labeling is called edge antimagic. total (vertex antimagic total). If a labeling is simultaneously edge-antimagic total and vertex antimagic total it is called a totally antimagic total labeling. A graph that admits totally antimagic total labeling is said to be totally antimagic total graph. Given totally antimagic total labeling f of G then, the function f(x) is such that f(x)= |V (G) + |E(G)| + 1 - f(x) for all X ϵ V (G) U E(G) is said to be complementary to f(x) or Complementary totally antimagic total labeling (CTAT). In this paper we establish complementary totally antimagic total labeling of some family of graphs Join G + K1, Paths Pn, Cycles Cn, Stars S1;n, Double Stars Bm;n, Wheels Wn and Corana GonK1:

KEYWORDS: Edge Antimagic Total, Totally Antimagic Total Graph, Complementary Totally Antimagic Total Labeling.2010 Mathematics Subject Classification: Primary 05C78; Secondary 05C76.

INTRODUCTION

All graphs in this paper are finite and undirected without loops or multiple edges. The graph G has vertex set V(G) and edge set E(G) and let |V(G)| = p and |E(G)| = q. The other terminologies or notations not defined here can be found in [7]. A labeling of a graph G is any mapping that sends a certain set of graph elements: vertices, edges or vertices and edges to a certain set of positive integers. If the domain is the vertex set or the edge set respectively, the labeling is called a vertex labeling or an edge labeling respectively. If the domain is both vertex set and edge set then the labeling is called a total labeling. More precisely, a(p,q)-graphG is total labeling if there exists a bijective function f: V (G) U E(G)→ {1, 2,…,p+q}. Moreover if the vertices are labed with the smallest possible numbers ie f(V (G)) = {1, 2,...,p}, then the total labeling is said to be Super edge magic. This concept is defined in [3]. If the vertices are labeled with the biggest possible numbers.

ie f(V (G)) = {q + 1, q + 2,…, q + p} . Then the total labeling is called Super magic total or

Complementary super edge magic total. This is defined in [1]. More precisely given total labeling f of

G, the function f(x) such that f (x) = p + q + 1 - f(x) for every element x ϵ V (G) U E(G) is called

Super magic total or Complementary super edge total. The edge weight of an edge is the sum of the

label of the edge and the labels of the end vertices of that edge, while the vertex weight of a vertex is

the sum of the label of the vertex and the labels of all the edges incident with that vertex. In other

words, the edge-weight w_f (uv) of an edge uv of G under the labeling f defined by

$$w_f(uv) = f(u) + f(uv) + f(v)$$

and the vertex weight $w_f(v)$ of a vertex v of G under the labeling f defined by

$$w_f(v) = f(v) + \sum_{u \in N(v)} f(uv)$$

where N(v) is the set of open neighborhood of v. A labeling *f* is called complementary edge antimagic total (CEAT) (complementary vertex antimagic total (CVAT)) if all edge weights (vertex weights) are pair wise distinct. A graph that admits CEAT (CVAT) total labeling is called a CEAT (CVAT) graph. If the edge weights (vertex weights) are all equal then

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The total labeling is called complementary edge magic total (CEMT) (Complementary vertex magic total labeling (CVMT)).

Since all graphs are EAT and VAT, naturally we can ask whether there exist graphs possessing, a labeling that is simultaneously vertex antimagic total and edge-antimagic total. We will call such a labeling, a totally antimagic total graph (TAT graph). The definition of totally antimagic total labeling is a natural extention of the concept of total magic labeling. If moreover the vertices are labeled with the biggest possible labels then the labeling is reffered as complementary totally antimagic total labeling graph (CTAT graph). The complementary totally antimagic total labeling is an alternate valuation of TAT-labeling.

There are known characterizations of all EAT and VAT graphs. In [6] Millar, Phanalasy and Ryan proved.

Preposition1: All graphs are super EAT. Preposition2: All graphs are super VAT.

The complementary Super EAT (Super VAT) is an alternate valuation of Super EAT (Super VAT)

Preposition1^1: All graphs are Complementary super EAT.

Preposition2^1: All graphs are Complementary super VAT

We say that the labeling f is ordered if $w_f(u) \geq w_f(v)$ and sharp ordered $w_f(u) > w_f(v)$ holds for every pair of vertices (u,v) ϵ V (G) such that f(u) > f(v). A graph that admits a ordered (Sharp ordered) is called a ordered (Sharp ordered) graph. In [2] proved that the Paths, Cycles, Stars, double Stars, Wheels and a unian of regular totally antimagic total are totally antimagic total graphs.

In this paper we will obtain Complementary totally antimagic total labeling of some family of graphs such as Join $G + K_1$, Path P_n, Cycle C_n and Corana $GonK_1$:

JOIN GRAPHS

Let *G* U *H* denote the disjiont union of graphs G and H. The Join G+H of two disjoint graphs is obtained from their union *G* ᵁ *H* by adding new edges joining each vertex of V(G) to every vertex of V(H). Thus G + H has $p_1 + p_2$ vertices and $q_1 + q_2 + p_1p_2$ edges. This concept was defined by Zykov [5]. In this section we deal with a totally antimagic total labeling of $G + K_1$. According to preposition 1^1 we have that every graph G is complementary Super EAT. If there exists a complementary super EAT labeling of a graph G satisfying the additional condition that it is also ordered, we are able to prove that the Join $G + K_1$ is Complementary total antimagic total graph (CTAT).

Theorem1: Suppose G is an ordered Complementary super EAT graph. Then $G + K_1$ is a Complementary TAT graph.

Proof: According to the preposition 1^1, we have that every graph G is Complementary super EAT. Let g be an ordered complementary super EAT labeling of G. Then

$$V(G) = \{v_1, v_2, \ldots, v_p\}$$

such that $g(v_i) = q + i$ for $i = 1, 2, 3,\ldots, p$

Since g is ordered then $w_g(v_i) \geq w_g(v_{i+1})$ for $i = 1, 2,\ldots, p - 1$

Let $u \in V(G + K_1)$ not labeling to G

We define a new labeling f of $G + K_1$ such that $f(x) = g(x)$ where $x \in V(G) \cup E(G)$
$f(u) = 1$

$f(uv_i) = 2p - q + 1 - i$ for $i = 1, 2, 3, .., p$

It is easy to see that there exists a bijection

$f : V(G + K1) \cup E(G + K1) \rightarrow \{2p + q + 1, 2p + q,\ldots, 3, 2, 1\}$

For the vertex weight under the labeling *f* we get the following

$w_f(u) = f(u) + \sum_{i=1}^{p} f(uv_i)$

$w_f(u) = 1 + \sum_{i=1}^{p}(2p - q + 1 - i)$

$w_f(u) = \frac{(2p-q)(2p-q+1)}{2}$

For $i = 1, 2,\ldots, p$ we have

$w_f(v_i) = f(v_i) + \sum_{v \in N_{g(vi)}} f(vv_i) + f(uv_i)$

$w_f(vi) = g(vi) + \sum_{v \in NG(vi)} g(vvi) + 2p - q + 1 - i$
$w_f(v_i) = w_g(v_i) + 2p - q + 1 - i$

$w_f(v_{i+1}) > w_g(v_i) + 2p - q + 1 - i = w_f(v_{i+1})$.

Moreover, as g is a Complementary Super EAT labeling of G, we have

$w_f(vp) = g(vp) + \sum_{v \in NG(vp)} g(vvp) + (p - q + 1)$

$w_f(v_p) \geq 2p + 1 + \sum_{j=1}^{p-1}(p + q + 2 - j) + (p - q + 1)$

$w_f(v_p) \geq 3p - q + 2 + [(p-1)(p+2q+4)]/2$

$w_f(v_p) > w_f(u)$

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Thus, the vertex-weight are all distinct the edge-weights of the edges in E(G) under the labeling f are all distinct as g is an complementary. EAT labeling of G. More precisely, we have $\max w_f(e) = \max w_f(e)$ for every $e \in E(G)$

Moreover, as g is Complementary super, for the lower bound on the maximum edge-weight of $e \in E(G)$ under the labeling f, we get

$\max w_f(e) = \max w_g(e) \geq q + (p + q - 1) + (p + q)$

$\max w_f(e) = \max w_g(e) = 2p + 3q - 1$

For $i = 1, 2,\ldots, p$ we get

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$w_f(uv_i) = f(u) + f(uv_i) + f(v_i)$

$w_f(uv_i) = 1 + (2p - q + 1 - i) + g(v_i)$

$w_f(uv_i) = 1 + 2p - q + 1 - i + q + i$

$w_f(uv_i) = 2p + 2 < 2p + 3q - 1 \leq \max w_f(e)$, where $e \in E(G)$

Clearly the edge weight are distinct. Therefore f is a complementary TAT labeling of $G + K_1$.

In some cases if we consider a given graph G with corresponding non order complementary super EAT labeling, after applying the labeling method presented in theorem 1, the obtained labeling can be also complementary TAT. However also when the obtained labeling is not complementary TAT it seems to be very easy to modify the obtained labeling by changing some labels and to get a complementary TAT labeling of $G + K_1$. Thus we state below conjecture.

Conjecture1: Every graph $G + K_1$ is complementary TAT. Weaker version of the conjecture 1.

Conjecture2: Every complete graph is Complementary TAT.

DISJOINT GRAPHS

Let mG denote the disjoint union of m copies of graph G. In this section we obtain some families of graphs are complementary TAT such as totally disconnected graphs of m vetices ie mK_1, m copies of K_2 ie mK_2, paths P_n, $n \geq 2$, Cycles C_n, $n \geq 3$, Wheels W_n, $n \geq 3$, friendship graphs F_n, $n \geq 1$, Fans F_n, $n \geq 2$ and Stars S_n, $n \geq 1$.

Furthermore, the complementary TAT labeling of these graphs have useful extra properties.

Lemma1: For every positive integer m the graph mK_1, $m \geq 1$ is sharp ordered complementary super TAT.

Proof: Trivial.

Lemma2: For every positive integer m the graph mK_2, $m \geq 1$, is sharp ordered complementary super TAT.

Proof: Let the vertices of mK_2 be v_i, $i = 1, 2, \ldots, 2m$ such that its edge set is $\{v_1v_2, v_3v_4, \ldots, v_{2m-1}v_{2m}\}$

Define the labeling g of mK_2 as follows

$g(v_i) = 3m + 1 - i$ for $i = 1, 2, \ldots, 2m$

$g(v_iv_{i+1}) = m + \frac{1-i}{2}$ for $i = 1, 3, \ldots, 2m - 1$

The edge weight of the edge v_iv_{i+1} is

$w_g(v_iv_{i+1}) = g(v_i) + g(v_iv_{i+1}) + g(v_{i+1})$ for $i = 1, 2, \ldots, 2m - 1$

$w_g(v_iv_{i+1}) = (3m + 1 - i) + (m + \frac{1-i}{2}) + (3m + 1 - (i + 1))$

$w_g(v_iv_{i+1}) = 7m - \frac{5i-3}{2}$

and for the vertex weight we get

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$$w_g(v_i) = \begin{cases} 4m - \dfrac{3(i-1)}{2}, & i = 1, 3, \ldots, 2m-1 \\ 4m - \dfrac{3(i-4)}{2}, & i = 2, 4, \ldots, 2m \end{cases}$$

Once can easily see that g is a complementary super TAT.

Lemma3: The path P_n, $n \geq 2$, is sharp ordered complementary super TAT

Proof: Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ such that

$E(P_n) = \{v_1v_2, v_2v_3, \ldots, v_{n-1}v_n\}$

One can easily check that the labeling $g : V(P_n) \cup E(P_n) \rightarrow \{2n-1, 2n-2, \ldots, 2, 1\}$ satisfies the complementary super TAT when

$$g(v_i) = \begin{cases} 2(n-i)+1, & i = 1, 2, \ldots, \left\lceil \frac{n}{2} \right\rceil \\ 2(i-1), & i = \left\lceil \frac{n}{2} \right\rceil + 1, \left\lceil \frac{n}{2} \right\rceil + 2, \ldots, n \end{cases}$$

$$g(v_i v_{2+1}) = \begin{cases} n+1-2i, & i = 1, 2, \ldots, \left\lceil \frac{n}{2} \right\rceil \\ 2i-n, & i = \left\lceil \frac{n}{2} \right\rceil + 1, \left\lceil \frac{n}{2} \right\rceil + 2, \ldots, (n-1). \end{cases}$$

Lemma4. The Cycle $C_n, n \geq 3$ is sharp order complementary super TAT

Proof. Let $V(C_n) = \{v_1, v_2, \ldots, v_n\}$ and $E(G) = \{v_1v_2, v_2v_3, \ldots, v_n v_1\}$

Consider the labeling $g : V(C_n) \cup E(C_n) \rightarrow \{2n, 2n-1, \ldots, 2, 1\}$ defined by

$$g(v_i) = \begin{cases} 2n, & i = 1 \\ 2n+3-2i, & i = 2, 3, \ldots, \left\lfloor \frac{n}{2} \right\rfloor + 1 \\ 2(i-1), & i = \left\lfloor \frac{n}{2} \right\rfloor + 2, \left\lfloor \frac{n}{2} \right\rfloor + 3, \ldots, n \end{cases}$$

$$g(v_1 v_{2+1}) = \begin{cases} n+2-2i, & i = 1, 2, \ldots, \frac{n}{2} \\ 2i-n-1, & i = \left\lfloor \frac{n}{2} \right\rfloor + 2, \ldots, (n-1). \end{cases}$$

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$g(v_n v_1) = n - 1$

It is easy to see that g is a sharp ordered Complementary super TAT labeling of C_n with required properties

Corollary1: The wheel $W_n = C_n + K_1$, $n \geq 3$ is Complementary TAT

Corollary2: The friendship graph $F_n = nK_2 + K_1$, $n \geq 1$ is Complementary TAT

Proof: The result follows from Lemma 2 and Theorem 1

Corollary3: The Fan $F_n = P_n + K_1$, $n \geq 2$ is complementary TAT

Corollary4: The Star $S_n = \overline{K_{n,}} + K_1$, $n \geq 1$ is complementary TAT

Proof: The result follows from Lemma 1 and Theorem 1

CORONA GRAPH

A Cycle of order m with n pendent edges attached at each vertex is Corona graph (Crown graph) $C_m$on K_1. Corona graph definition is defined in [4]

Theorem2: Let G be regular ordered Complementary super EAT graph. Then Corona GonK_1 is Complementary TAT.

Proof: Let g be an ordered Complemenary Super EAT labeling of regular graph G. Since g is complementary super EAT, let the vertices of G be $v_1, v_2, \ldots, v_p$ such that

$g(v_i) = pn + 1 - i$ for $i = 1, 2, \ldots, p$

and $w_g(v_i) \geq w_g(v_{i+1})$ for $i = 1, 2, \ldots, p$

Denote the vertices other than the vertices of G by $u_{i,j}$; $i = 1, 2, \ldots, p$, $j = 1, 2, \ldots, n$ such that $u_{i;j}v_i \in E(GonK_1)$

Let the function $f : V(GonK_1) \rightarrow \{(n + 1)p, (n + 1)p - 1, \ldots, 2, 1\}$
be defined by

$f(x) = g(x)$ where $x \in V(G) \cup E(G)$

$f(u_{i,j}) = (3p + 1)n + 1 - ni - j$, $i = 1, 2, \ldots, p$; $j = 1, 2, \ldots, n$

$f(u_{i;j}v_i) = (2p + 1)n + 1 - ni - j$, $i = 1, 2, \ldots, p$; $j = 1, 2, \ldots, n$

Clearly we can see that

$f : V(GonK_1) \cup E(GonK_1) \rightarrow \{p + q + 2pn, p + q + 2pn - 1, \ldots, 2, 1\}$.
Edge weight of GonK_1

$w_f(v_i v_k) = f(v_i) + f(v_k) + f(v_i v_k)$, $i = 1, 2, \ldots, p$; $k = 1, 2, \ldots, p$; $i \neq k$

$w_f(v_i v_k) = w_g(v_i v_k)$

As g is a Complementary EAT, the weights of all the edges $e \in E(G)$ are different under the labeling f as well. For the edge

$u_{i;j}v_i$, $i = 1, 2, \ldots, p$; $j = 1, 2, \ldots, n$ we get

$w_f(u_{i;j}v_i) = f(u_{i;j}) + f(u_{i;j}v_i) + f(v_i)$

$w_f(u_{i;j}v_i) = (3p + 1)n + 1 - ni - j + (2n + 1)n + 1 - ni - j + g(v_i)$

$w_f(u_{i;j}v_i) = (3p + 1)n + 1 - ni - j + (2n + 1)n + 1 - ni - j + (pn + 1 - i)$

$w_f(u_{i;j}v_i) = (6p + 2)n + 3 - (2n + 1)i - 2j$; $i = 1, 2,\dots, p$; $j = 1, 2,\dots, n$

One can easily see that all the edge in $E(GonK_1)$ |$E(G)$ have different edge-weights. Moreover

$w_f(u_{i;j}v_i) = (6p + 2)n + 3 - (2n + 1)i - 2j$

$w_f(u_{i;j}v_i) \geq (6p + 2)n + 3 - (2n + 1)p - 2n = 4pn - p + 3$

$w_f(u_{i;j}v_i) > 3(pn + 1)$; $i = 1, 2,\dots, p$; $j = 1, 2,\dots, n$

Also $w_f(e_1) < w_f(e_2)$ where $e_1 \in E(G)$ and $e_2 \in E(GonK_1)|E(G)$

Thus edge weights of all the edges in $GonK_1$ are distinct.

Also we have to verify that the vertex-weights are all distinct.

For the vertex $u_{i;j}$; $i = 1, 2,\dots, p$; $j = 1, 2,\dots, n$, we have

$w_f(u_{i;j}) = f(u_{i;j}) + f(u_{i;j}v_i)$

$w_f(u_{i;j}) = [(3p + 1)n + 1 - ni - j] + [(2p + 1)n + 1 - ni - j]$

$w_f(u_{i;j}) = (5p + 2)n + 2 - 2ni - 2j$

Thus the vertex weights are all different numbers from the set $\{5pn, 5pn - 2,\dots,3pn + 4\}$

For the vertex v_i; $i = 1, 2,\dots, p$ we have

$w_f(v_i) = \sum_{j=1}^{n} f(u_{i,j}v_i) + f(v_i) + \sum_{v_iv_k \in E(G)} f(v_iv_k)$

$w_f(v_i) = \sum_{j=1}^{n} [(2p + 1)n + 1 - ni - j] + g(v_i) + \sum_{v_iv_k \in E(G)} g(v_iv_k)$

$w_f(v_i) = [(2p + 1) n^2_j + n - n^2_j - \sum_{j=1}^{n} + [g(v_i) + \sum_{v_iv_k \in E(G)} g(v_iv_k)$

$w_f(v_i) = - [n(n+1)]/2+(2pn+n+1)n+w_g(v_i) - n^2 i$

$W_f(v_i) < 5pn^2$

As $w_g(v_i) \leq w_g(v_{i+1})$ for $i = 1, 2,\dots, p - 1$, this shows that also under the labeling f the vertex weights are all distinct.

Obviously from Theorem 2 and Lemma 2 and 4 we get the following Corollaries

Corollary5. The bistar $B_{n,m} \simeq K_{2on} K_1$, $n \geq 1$ is complementary TAT

Corollary6: The n-crown $C_m onK_1$, $n \geq 1$, $m \geq 3$ is Complementary TAT

Immediately from Theorem 2 and Lemma 1 we get that the Star $S_n = K_1 onK_1$, $n \geq 1$ is Complementary TAT. Also as in the proof of Theorem 2 and using Lemma 3, we can also prove the following Corollary 7
Corollary7: The connect m Stars $P_m onK_1$, $m \geq 2$, $n \geq 1$ is Complementary TAT.

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